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Improving prospective mathematics teachers' reversible thinking ability through a metacognitive-approach teaching

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Abstract

Reversible thinking ability is an individual ability to do a cognitive process by reverse action, contributing to one of the student's competencies in solving mathematical problems. Many students encounter difficulties in solving problems that require reversible thinking due to the limitedness of teachers' proficiency in mastering this ability. Different studies have suggested various teaching approaches to improve this ability in teacher education; however, teaching with a metacognitive approach has not yet been addressed in the recent study. Therefore, this study aims to improve prospective teachers' reversible thinking ability through metacognitive-approach teaching. A quasi-experimental method with a pre-test, intervention, and post-test design was used in this study. The participants were 118 prospective mathematics teachers at two universities in Bandung, Indonesia, divided into two groups: 58 were in the experimental group, and the rest were in the control group. The participant's initial mathematical ability (IMA) in both groups was categorized into high, moderate, and low. Using t-test, Mann-Whitney test, and Kruskal-Wallis test, the result shows a meaningful difference in the improvement of reversible thinking ability between prospective teachers who received the metacognitive-approach teaching and those who did not. However, there is no significant interaction effect between the teaching approach and initial math ability on the improvement of reversible thinking ability. To conclude, the recent study's findings revealed that the metacognitive approach effectively improves prospective teachers' reversible thinking in all IMA levels. Thus, it needs to be considered one of the alternative teaching approaches in higher education, especially in teacher education.

Keywords: reversible thinking, metacognitive approach, prospective mathematics teacher, quasi-experimental method

INTRODUCTION

The individual cognitive development theory, proposed by Piaget (2013), states that each person will go through four stages of cognitive development in their lifetime. According to Piaget (2013), individuals learn about their environment through sensory and motor skills and can perform activities with reflexes in their first stage of development (Pogozhina, 2018). The second, the pre-operational stage, is characterized by a thinking process centered on one situation. The thinking process continues to develop until, at a later stage, individuals can preserve a condition or conservative nature (Piaget, 2013). At this stage, the ability to think reversibly begins to emerge.

Reversible thinking ability, one of the central units in the development of thinking processes first introduced by Piaget (2013), is an individual ability to perform cognitive activities in a reversible order to solve a problem (Furth et al., 1976; Maf'ulah & Juniati, 2021; Maf'ulah et al., 2016; Oakley, 2004; Ramful, 2014, 2015). Krutetskii et al. (1976) observed and associated the reversible thinking process with the thinking process in mathematics. Many mathematical objects require students to solve problems by working backward: a method in the solution proposed by Polya (2004), where one has to reverse a mental action to return from the

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Contribution to the literature

- This study reported that implementing the metacognitive approach can significantly improve prospective teachers' reversible thinking ability. This is significant because the study has proven that the metacognitive approach is an effective strategy to teach mathematics, not only to school students, as prominently investigated by existing studies but also to prospective elementary mathematics teachers.
- This study's findings will allow the readers to gain new insight into how this teaching approach is statistically proven to improve one's reversible thinking.
- This study can be applied to learners in all mathematics ability levels (low, moderate, and high). Likewise, this study can be one of the factors considered when deciding the practical lecture-based teaching approach for elementary mathematics education.

result of a process to the start of the process (Ramful, 2015). Working in reverse or backward is usually associated with complex problems or high-level problems whose solution requires a problem-solving thinking process. Therefore, reversible thinking affects the success of other problem-solving mathematical competencies (Maf'ulah & Juniati, 2020a). According to experts, solving problems is an essential aspect of mathematics that students must master because it is the core of mathematics learning (Maf'ulah & Juniati, 2020b; Wikström et al., 2020).

Despite the emergence of reversible thinking ability in the success of solving mathematics problems, studies that reveal students' weaknesses in acquiring this ability are prominent (Hackenberg, 2010; Ikram et al., 2018). For example, students find it easy to determine $f(3)$ from $f(x)=x^3-x$, however, they possessed difficulties to determine $f^{-1}(x)=3$ since finding the original function from its inverse is usually not a trivial task given to students (Simon et al., 2016). Indeed, the inverse function is highly associated with a reverse action, yet many students proposed the wrong ways to determine the rule for $f^{-1}(x)$ (Carlson et al., 2015; Ikram et al., 2020). However, paying attention to how the teachers teach is also essential. Students' weaknesses in reversible thinking are also associated with the tasks given by teachers during the learning activity. Hiebert and Wearne (1993) stated that knowledge gained by students is highly determined by the tasks given to them. Therefore, teachers are responsible for familiarizing students with tasks whose solutions require a reverse mental action to nurture students' reversible thinking.

The study was initiated on the empirical fact that the reversible thinking of prospective mathematics teachers exists as an issue (Maf'ulah & Juniati, 2019, 2020a; Paoletti et al., 2018). According to Maf'ulah and Juniati (2020a), only 8.3% of all participating students correctly answered when asked to determine the value of m in a given algebraic equation. The possible factor was that they did not involve their reversible thinking, which was, rechecking the result by substituting m to the initial equation. Another problem was investigated by Maf'ulah and Juniati (2019) to 105 prospective mathematics teachers focused on the reversible relationships between two function representations: graphical and symbolic. The result revealed that most students could draw a graph from the given function; meanwhile, few could define a function from the given graph.

Furthermore, a preliminary study was conducted to examine elementary prospective teachers' reversible thinking. In this study, student teachers were given to perform the following task:

> If Simon takes three days to paint a room while Ethan takes 6 days to paint the room, how many days are needed if they paint the room together?

Unfortunately, a few students worked on the following solution:

> If Simon takes three days/room, he paints 1/3 room per day. With the same strategy, Ethan paints 1/6 room per day. Thus, they will paint (1/3+1/6=3/6) room per day, so it will take them two days to paint the room.

Instead, many student teachers simplified the solution to (3+6)/2, resulting in an incorrect answer of 4.5 days. Drawing from existing research and recent investigation on prospective teachers' reversible thinking, the problems found emerge the urgency to find the solution. Students' reversible thinking can be developed through learning mathematics, which is indisputably part of the teacher's responsibility. If the teachers are responsible for developing students' reversible thinking ability, then developing or prospective teachers' reversible thinking ability is worth the concern.

To improve the reversible thinking ability of prospective mathematics teachers, paying attention to possible factors is essential to accomplish this objective. One of these factors is the teaching approach used by university lecturers. Teaching with a direct approach is the most popular and well-known for its simplicity among lectures. However, demonstrating a math concept with a direct approach does not promote students' reversible thinking since it is central to the teacher-student direct communication, rehearsal, and memorization procedure (Ewing, 2011; Stein et al., 1996). Therefore, another suitable teaching approach is needed to ensure prospective teachers foster their awareness and monitor every step they have chosen to solve math problems. Deepening specialized mathematical knowledge for teaching, such as understanding and developing mathematics procedures, empowers the development of pedagogical content knowledge for prospective elementary mathematics teachers (Kajander & Holm, 2016).

A metacognitive approach may offer an opportunity for redesigning prospective teachers' activity during the lectures. Metacognition, also known as thinking about thinking, is a higher-level thinking process that involves self-control of cognitive process (Mevarech, 2014). In teaching with a metacognition approach, most educators would include strategies that equip students to plan, monitor, evaluate, and control their performance while completing a task (Perry et al., 2019). The most important aspect is that students can employ these strategies in a conscious, controlled manner to solve the problem. Moreover, several studies empirically proved that a metacognitive approach has successfully improved students' academic performance (Perry et al., 2019). A significant outcome of metacognition found in teaching mathematics (Dignath et al., 2008; Sahin & Kendir, 2013), science (Zohar & Barzilai, 2013), language (Yang et al., 2021), and cross-curricular (Mannion & Mercer, 2016; Perry et al., 2012).

Nevertheless, few studies have been concerned with the impact of the metacognitive approach in teaching prospective teachers. Although it has been stated that metacognition can be taught successfully at the university level (der Stel & Veenman, 2008, 2010; Veenman et al., 2006), further investigation is required to determine how the teaching approach can contribute to the performance of prospective mathematics teachers, particularly how it can help them cultivate reversible thinking when solving math problems. Therefore, in this study, the researcher will investigate how the metacognitive-approach teaching in the elementary mathematics course can contribute as an alternative teaching approach to foster prospective mathematics teachers' reversible thinking ability.

THEORETICAL FRAMEWORK

What is Reversible Thinking Ability?

The thought process of finding solutions to mathematical problems by reversing the sequence of occurrence or returning the direction of thinking back to the starting point is called a reversible thinking process (Olive & Steffe, 2001; Saparwadi et al., 2020). In other words, reversible thinking is a cognitive activity in finding solutions to problems when the final results are known and asked to find initial conditions. Therefore, students might have a comprehensive conceptual understanding to make a good connection between concepts to solve a problem in two ways. Solving problems sequentially by working backward means that students can solve problems in forward-thinking process.

Piaget and Duckworth (1970) conceptualized reversible thinking into two indicators: negation and reciprocity. Negation displays the idea that every direct operation can be canceled. Notably, every direct operation has an inverse. For instance, the addition operation is canceled by the subtraction operation. The other indicator, reciprocity, presents the relational structure of an equation or inequality. For example, the expression 1+1=2 can be perceived as a collection of objects in the right segment and a composition of objects in the left segment.

Furthermore, reversible thinking is required in perceiving that the whole is the fusion of each component, and conversely, each part will connect and synthesize the whole. This part-whole scheme can be examined in the fraction domain as conducted in this study. For example, *a rectangular shape is* $2/3$ *of a shape; what is the shape as a whole?* Likewise, Tzur (2004) conceptualized reversibility in the domain of fractions. He visualized the conception of n/m as a particular unit relative to a whole given unit. To illustrate this, *the half of a triangle from a piece of wood is the same size as the half of a rectangle from a piece of wood of the same size*. Usually, this treatment will make students mistaken that half of the triangle is bigger than half of the rectangle.

Metacognitive Approach

One of the learning strategies that can accommodate students' reversible thinking process is the metacognitive approach (Nurkaeti et al., 2019). Metacognitive strategies will stimulate students to think of alternative strategies for solving mathematical problems (Tachie, 2019). Metacognitive strategies can also encourage students to find, think, compare, and even predict possibilities in future conditions (Bakar & Ismail, 2019). Furthermore, Waskitoningtyas (2015) explained that metacognitive strategies could design, monitor, and control what students know, what students need to do, and how to do it so that students realize when they understand and when they do not understand concepts in learning.

Metacognition alludes to students' awareness of their abilities, i.e., the ability to comprehend, regulate, and manipulate cognitive processes. According to Hewitt (2008), metacognition is the ability to ask and answer questions such as, *what do I know about this topic? Do I have the required knowledge? Do I know where I can find the required information? Which strategies and tactics should be employed?* Thus, teaching and learning with a metacognitive approach are designed to integrate metacognitive questions related to the topic being studied and the control of the thinking process.

Research Question

Based on the explanation above, this study seeks to improve the reversible thinking ability of prospective mathematics teachers through metacognitive-approach teaching. Two main questions are proposed:

1. Is there any difference in the improvement of reversible thinking ability between prospective teachers who learn with a metacognitive approach and those who learn with a direct approach?
2. Is there any interaction effect between the implementation of the teaching approach (metacognitive approach and direct approach) and prospective teachers' initial mathematics ability on the improvement of reversible thinking ability?

Following research questions, two hypotheses were formulated:

1. There is a significant difference in the improvement of reversible thinking ability between prospective teachers who learn with a metacognitive approach and those who learn with a direct approach
2. There is an interaction effect of the teaching approach (metacognitive approach and direct approach) and prospective teachers' initial mathematics ability on the improvement of students' reversible thinking ability.

METHOD

Research Design

The study employed a quasi-experimental method with a pre-test, intervention, and post-test design to investigate the impact of the metacognitive teaching approach in improving prospective teachers' reversible thinking ability.

Study Sample

The sample of this study was prospective elementary school mathematics teachers who attended mathematics education courses at two different universities in Bandung, West Java, Indonesia. The sample was selected by stratified random sampling (Freud & Wilson, 2003), and 118 prospective mathematics teachers were chosen (19 to 21 years old on average). All participant was separated into two groups, that was the experimental and control groups. During the study, the experimental group consisted of 58 prospective teachers. In contrast, 60 prospective teachers were in the control group. Both groups were studying at two campuses to ensure the result's objectivity during the teaching experiment. For all prospective teachers, Indonesian was their first language.

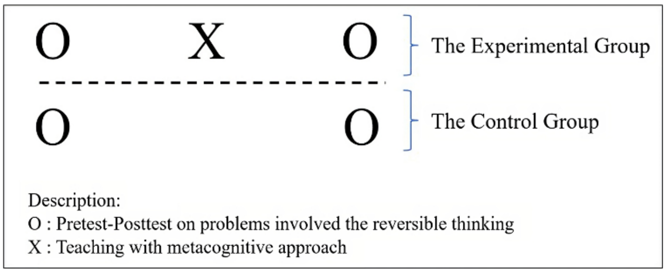

Figure 1. Pre-test, intervention, & post-test design using quasi-experimental method (Source: Author's own elaboration)

Intervention

An intervention was designed to measure the impact of the metacognitive approach on prospective teachers' reversible thinking ability. During the study, teachers in the experimental group received a metacognitive approach, while teachers in the control group received learning with a direct approach. Different teaching approaches were intended to examine whether there is any significant difference in the improvement of prospective teachers' reversible thinking ability between those who experienced the metacognitive approach and direct-approach teaching. Before the teaching experiment, both groups were tested to assess their initial mathematical ability (IMA). The test result categorized participants' mathematical ability into high, moderate, and low in respective groups.

For each group, eight sessions were administered for the intervention. The introductory session was intended to explain the purpose of the teaching experiment, ensuring that each prospective teacher understood the teaching process and the research instruments used. Following that, each group was assigned an IMA test and a pre-test about reversible thinking problems on the topic of fractions. The second to seventh meetings were allocated to do the teaching treatment, the metacognitive approach for the experimental group, and the direct approach for the control group. The last session was for the post-test. Each session lasted for 150 minutes. Likewise, these sessions were conducted during regular course study hours. The intervention is depicted in **Figure 1**.

Furthermore, the study was also intended to investigate the interaction effect between the teaching approaches (metacognitive and direct) and prospective teachers' IMA to improve their' reversible thinking ability. **Table 1** describes the codes for each interrelationship between factors applied in this study.

Data Collection

The research instruments developed include data collection instruments and learning devices. The

Table 1. Interrelationships between factors

Teaching approach	(B_1)	(B_2)
IMA level		
(A_1)	A_1B_1	A_1B_2
(A_2)	A_2B_1	A_2B_2
(A_3)	A_3B_1	A_3B_2

Note. IMA: Initial mathematical ability; B_1 : Metacognitive-approach teaching/experimental group; B_2: Direct-approach teaching/control group; A_1: IMA in high level; A_2: IMA in moderate level; A_3: IMA in low level; A_iB_j: Score of students with initial mathematical ability i who received teaching approach j ($i = 1, 2, 3; j = 1, 2$).

Task 1. Explain the meaning of the following fraction multiplications and write down your method to find the result.

a. $\frac{1}{2} \times 4$

b. $9 \times \frac{1}{3}$

c. $\frac{1}{2} \times \frac{1}{3}$

Task 2. Explain the meaning of the following fraction divisions and write down your method to find the result.

a. $\frac{1}{2} \div 2$

b. $3 \div \frac{1}{3}$

c. $\frac{1}{2} \div \frac{1}{6}$

Figure 2. Questions to examine prospective teachers' IMA in multiplication & division of fractions (Source: Author's own elaboration)

instrument consisted of a test to assess the prospective teachers' IMA and a pretest-posttest of the problem, where its solution requires reversible thinking ability. Since the participants were prospective primary mathematics teachers, all the problems assigned covered the topic of fractions, a mathematics topic learned in primary schools. IMA test was addressed to examine prospective teachers' initial understanding of division and multiplication of fractions, comprised of six questions. The first three questions assessed the meaning of fraction multiplication, while the last three questions were on fractions' division, as described in **Figure 2**.

It is common in Indonesia to define multiplication as repeated addition and division as repeated subtraction. The conceptions are only sufficient for integers' operation; however, they could raise a problem in the case of fractions. A good understanding of those two operations that can be true for real numbers is multiplication as *of* or *part of*, while divisions as *the inverse of multiplication*.

For instance, in **Figure 2**, task 1 (a) means *one-half of four* rather than *adding one-half four times*. Therefore, it was essential to identify prospective teachers' IMA before investigating their reversible thinking ability. Additionally, the IMA test was performed by participating students for about thirty minutes.

Task 1 (Example question for negation indicator)

Mr. Andy would take 12 days to build a fish pond. If he build the pond together with his son, David, it would take four days. How many days needed for David if he build the pond by himself?

Task 2 (Example question for reciprocity indicator)

Water hyacinth, or known as *Eceng Gondok*, is a floating aquatic plant commonly found in several areas in Indonesia, especially in ponds. If this plant multiplies as twice as the previous amount in one day and will cover the whole pond's surface in 60 days, how many days are needed for the plant to cover a quarter of the pond's surface?

Figure 3. Example of a question for pre- & post-test (problem given is in topic of fractions whose solution needs a reversible thinking ability) (Source: Author's own elaboration)

Furthermore, the reversible thinking problems developed were based on predetermined indicators, namely:

(1) Negation, i.e., the use of the inverse of related operations in making equations, and

(2) Reciprocity, i.e., the use of equivalent relationships with given equations.

The example questions from the pre-test and post-test are specified in **Figure 3**.

Commonly, suppose the student is accustomed to the direct-solution strategy. In the case of task 2 (**Figure 3**), the student will answer, as follows:

> If the plant covers the pond's surface in 60 days, then it will cover a quarter of the pond's surface in 15 days since 60 is divided by 4 is 15.

However, if we read carefully, *the plant grows twice its amount in a day*, and if 15 days are required to fill a quarter of the pond's surface, then half of the pond's surface will be filled by the plant on day-16, and it will be fully covered in day-17, not day-60. Nonetheless, suppose the student is aware of employing a reversible-solution strategy. In that case, the student will start from the final result and work backward to find the initial condition:

> If the plant covers the pond's surface in 60 days, then it will cover half of the pond's surface in 59 days and a quarter of the pond's surface in 58 days.

The test instrument was tested on 40 prospective teachers (outside the sample) to see the suitability of the indicators and the items, the clarity of the language used, the feasibility of the items, and the correctness of the material or concepts tested. The learning tools developed include lecture units, teaching materials, worksheets describing the teaching approach, and the addressed mathematical competencies.

Data Analysis

Data analysis was aimed at testing the hypotheses proposed. The data for this study were then analyzed using statistical package for the social sciences (SPSS) software. Before data analysis was carried out, it was necessary to ensure that the assessment of prospective teachers' answers, especially test results, had been carried out objectively following the established criteria. For this reason, two assessors competent in providing mathematics education lectures were involved in examining the prospective teachers' test answers.

There are five packages of prospective teachers' test answers: the answers of IMA's test, the experimental group's pre-and post-test, and the control group's pre- and post-test answers. To test the hypotheses proposed, this study conducted three phases of data analysis.

Phase 1: Testing IMA score difference between experimental and control groups

As the initial phase of the analysis, testing IMA scores' differences between the experimental and control groups was conducted using the t-test or Mann-Whitney tests. It was necessary to test the normality and homogeneity of both data (experimental and control) before testing IMA score difference. The normality test was done using Shapiro-Wilk test, while the homogeneity was examined by Levene test, each at the significance level $\alpha = 0.05$. Suppose the normality and homogeneity tests of both data result at the sig. value $>\alpha$, IMA score different test is conducted using t-test. Otherwise, if one test result exists, in the normality or homogeneity of each group data, at the sig. value $<\alpha$, then Mann-Whitney test is used to examine the difference between IMA score of both groups.

Furthermore, the following hypothesis was proposed to test IMA score difference between the experimental and control group.

H_0: There does not exist a difference in the IMA score between the experimental and control groups.

H_1: There exists a difference in the IMA score between the experimental and control groups.

If the test, with t-test or Mann-Whitney, obtains sig. value $>\alpha = 0.05$, then H_0 is accepted, otherwise, H_0 is rejected (H_1 is accepted).

Phase 2: Testing reversible thinking test score difference between pre- and post-test

The second phase of analysis was done to assess if there is any significant improvement of prospective teachers' reversible thinking ability between teachers who were taught using the metacognitive and direct approach. To reach this aim, three sequential tests were performed, as follows:

(1) testing the normality and homogeneity of pre- and post-test, both experimental and control groups,

2) testing pre-test score between experimental and control groups, and

3) testing post-test score between experimental and control groups.

The first test was done with the same analysis as the first phase; meanwhile, the second and third tests can be performed with t-test or Mann-Whitney test depending on the result of the first test. Furthermore, the following hypothesis was proposed to examine the pre-test score difference between the experimental and control group. Notice that the t-test is used if the normality and homogeneity of the data are satisfied; otherwise, the Mann-Whitney test is used.

H_0: There does not exist a difference in the pre-test score between the experimental and control groups.

H_1: There exists a difference in the pre-test score between the experimental and control groups.

If the test results sig. value $>\alpha = 0.05$, then H_0 is accepted, otherwise, H_0 is rejected (H_1 is accepted).

The hypothesis was also proposed to examine the post-test score difference between the experimental and control groups.

H_0: There does not exist a difference in the post-test score between the experimental and control groups.

H_1: There exists a difference in the post-test score between the experimental and control groups.

If the test results sig. value $> \alpha = 0.05$, then H_0 is accepted, otherwise, H_0 is rejected (H_1 is accepted).

The final decision was made after comparing the result of those tests. Suppose H_0 in (2) is accepted and H_0 in (3) is rejected. In that case, it can be concluded that there is a significant difference in the improvement of reversible thinking ability between prospective teachers who learn with a metacognitive approach and those who learn with a direct approach.

Phase 3: Testing interaction effect of teaching approach and prospective teachers' IMA level on improving their reversible thinking ability

The final analysis phase was done to assess the interaction effect of the teaching approach (metacognitive and direct) and prospective teachers' IMA level on improving their reversible thinking ability. To achieve this aim, the following three consecutive tests were conducted:

(1) testing normality and homogeneity of the gain score in both the experimental and control groups,

(2) testing the gain score between low, moderate, and high IMA levels in the experimental group, and

(3) testing the gain score between low, moderate, and high IMA levels in the control group.

The first test was conducted using the same analysis as the first phase; the second and third tests will be conducted using either two-way ANOVA or Kruskal-

Wallis test, depending on the outcome of the first test. Furthermore, the two-way ANOVA test is performed if the normality and homogeneity of each data in (1) are met; otherwise, the Kruskal-Wallis test is used. The following hypothesis was proposed to examine the Gain score in (2).

H_0: There does not exist a difference in the reversible thinking test's gain score between low, moderate, and high IMA levels in the experimental group.

H_1: There exists a difference in the reversible thinking test's gain score between the low, moderate, and high IMA levels in the experimental group.

If the test results sig. value $>\alpha = 0.05$, then H_0 is accepted, otherwise, H_0 is rejected (H_1 is accepted).

The hypothesis was proposed to examine the Gain score in (3). as follows.

H_0: There does not exist a difference in the reversible thinking test's gain score between the low, moderate, and high IMA levels in the control group.

H_1: There exists a difference in the reversible thinking test's gain score between the low, moderate, and high IMA levels in the control group.

If the test results sig. value $>\alpha = 0.05$, then H_0 is accepted, otherwise, H_0 is rejected (H_1 is accepted).

The overall decision was made after evaluating the results of these tests. Suppose H_0 in (2) and (3) is rejected. In that case, it can be concluded that the teaching approach and IMA levels have a significant interaction effect on improving the prospective teacher's reversible thinking ability.

RESULTS

As mentioned, the study sought to examine the improvement of prospective mathematics teachers' reversible thinking ability through metacognitive-approach teaching. Three quantitative data analyses were conducted, namely

(1) analysis of IMA between prospective teachers in the control and experimental group,

(2) analysis of prospective teachers' reversible thinking between those who were taught using metacognitive and direct approaches, and

(3) analysis of the interaction effect of teaching approaches and IMA levels on the improvement of reversible thinking ability.

The following sections will present the findings of the study.

Data Analysis of Prospective Teachers' Mathematical Initial Ability

Before the teaching experiment, the analysis of prospective teachers' IMA of the experimental and control groups was conducted to ensure both groups' IMA were equivalent. It was intended to minimize the interference of variables other than the teaching approach used, such as the imbalance IMA of both groups, to result in reversible thinking ability improvement. Therefore, IMA test was assigned to all prospective teachers who participated before the implementation of the teaching. The statistical description of the experimental and control groups' IMA is depicted in **Figure 4**.

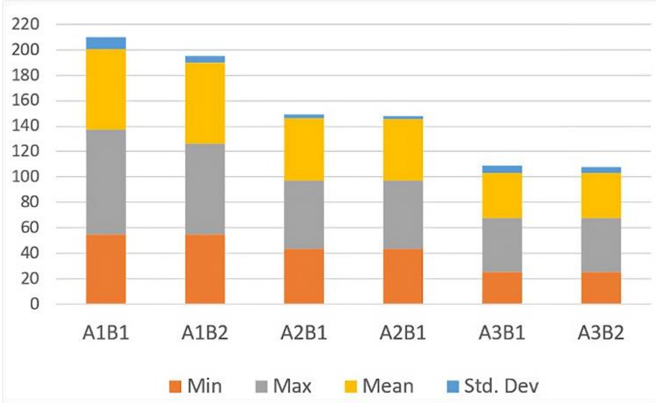

Figure 4. Descriptive statistics on IMA of experimental & control groups (Source: Author's own elaboration)

According to **Figure 4**, both groups have similar IMA scores. For instance, the mean scores of A_1B_1 (prospective teachers in the experimental group who have high IMA) and A_1B_2 (prospective teachers in the control group who have high IMA) are similar: $M = 63.162$ and $M = 63.529$, respectively. The following inferential statistics test was carried out to increase confidence in the outcome. The normality test using Shapiro-Wilk test on the experimental group data obtained a $\text{sig. value} = 0.043 < 0.05 = \alpha$, and on the control group, the sig. value obtained was $0.196 > 0.05 = \alpha$. The test indicates that at the significance level of $\alpha = 0.05$, the IMA's scores in the experimental group were not normally distributed, while in the control group, they were normally distributed.

Attributed to the fact that the experimental group's data distribution was not normally distributed, Mann-Whitney test was used to compare the average IMA scores of the experimental and control groups. The test obtained $\text{sig. value} = 0.0889 > \alpha = 0.05$, thus H_0 is accepted. The test implies that the IMA scores between both groups were not remarkably different at $\alpha = 0.05$. Correspondingly, the experimental group's IMA levels were relatively similar to those of the control group in high, moderate, and low levels.

Data Analysis of Prospective Teachers' Reversible Thinking Ability

The statistical result of both groups' IMA informs that the experiment will begin at the same starting point. The study continued to evaluate the improvement of the reversible thinking ability of prospective teachers.

Likewise, the experimental group received metacognitive-approach teaching, while the control

Table 2. Average pre- & post-test scores of experiment & control groups by IMA

Test in reversible thinking	Experiment group		Control group	
	Pre-test	Post-test	Pre-test	Post-test
IMA level				
Combined	37.723	56.267	36.207	42.229
Low	26.482	49.814	30.876	36.491
Moderate	39.200	55.734	33.334	40.405
High	47.451	63.529	46.667	51.667

group received direct-approach teaching. Moreover, the reversible thinking ability was evaluated using the pre- and post-test, which consisted of mathematical problems that require reversible thinking. The pre-and post-test were examined to both the experimental and control groups. In summary, the experimental and control groups' average pre- and post-test scores by IMA level are described in **Table 2**.

Furthermore, answers were also sought to investigate whether there is a difference in the improvement of reversible thinking ability between prospective teachers who were taught through a metacognitive approach and those who were taught through a direct approach based on IMA level. For that purpose, the difference between pre-and post-test scores for the experimental and control groups was determined first.

Tests were conducted on the experimental and control groups' general pre-and post-test scores, excluding IMA level of both groups (IMA level combined in **Table 2**). The pre-test and post-test data for the experimental and control groups were normally distributed and homogeneous. The t-test obtained the value of $sig. = 0.506 > 0.05 = \alpha$ for the pre-test score; thus, H_0 is accepted. The result implies no remarkable distinction in the pre-test scores of the experimental and control groups. As for the post-test data, the value was $sig. = 0.000 < 0.05 = \alpha$, therefore, H_0 is rejected. Unlike the pre-test, there was a remarkable difference in post-test scores of the experimental and control groups.

The test results suggest a significant difference in the prospective teachers' reversible thinking ability who received treatment with a metacognitive approach and those with a direct approach. This statement is based on the results of statistical tests, which state that:

(1) there was no significant difference in pre-test between experimental and control groups and

(2) there was a significant difference in post-test between experimental and control groups.

Furthermore, by paying attention to the average post-test score of the experimental group and control groups in **Table 2**, it appears that the average post-test score of the experimental group was higher than the average post-test score of the control group. Therefore, the metacognitive-approach teaching to prospective mathematics teachers at the university leads to a more significant improvement in their reversible thinking abilities than the direct approach.

Table 3. Average gain score in reversible thinking ability by teaching approach & IMA level

Teaching approach	Metacognitive (M)	Direct (L)
	Experiment group	Control group
IMA level		
Low	23.334	5.616
Moderate	16.533	7.073
High	16.078	5.000

Data Analysis of Interaction Effect of Teaching Approaches and IMA Levels on Improvement of Reversible Thinking Ability

In addition to testing the effectiveness of the metacognitive-teaching approach on improving the reversible thinking ability of prospective mathematics teachers, Further analysis was performed to assess the interaction effect of the teaching approach and prospective teachers' IMA level on improving their reversible thinking ability. The improvement of prospective teachers' reversible thinking ability is measured using the gain score in the experimental and control groups. The average gain scores of both groups' reversible thinking abilities are shown in **Table 3**.

Table 3 shows that the average gain score of reversible thinking ability in all IMA levels in the experimental group is greater than in the control group. To test whether there is an interaction effect of the teaching approach and IMA level on the improvement of prospective teachers' reversible thinking ability, a two-way ANOVA test was used if the data were normally distributed. Nevertheless, the Shapiro-Wilk test obtained the value of $\text{sig.} = 0.000 < 0.05 = \alpha$, implying that the gain data for the experimental group's reversible thinking ability at a low IMA level was not normally distributed. A similar outcome happened for the moderate and high IMA levels with the $\text{sig. value} = 0.001 < 0.05 = \alpha$ (moderate) and $\text{sig. value} = 0.008 < 0.05 = \alpha$ (high). Under these conditions, the interaction effect analysis between the teaching approach and IMA level on improving prospective teachers' reversible thinking ability was carried out using Kruskal-Wallis test. An illustration of the interaction effect analysis of teaching approaches and IMA level on the improvement of prospective teachers' reversible thinking ability was presented in **Figure 5**.

Figure 5 reveals that the experimental group's gain score is higher than the control group. This condition

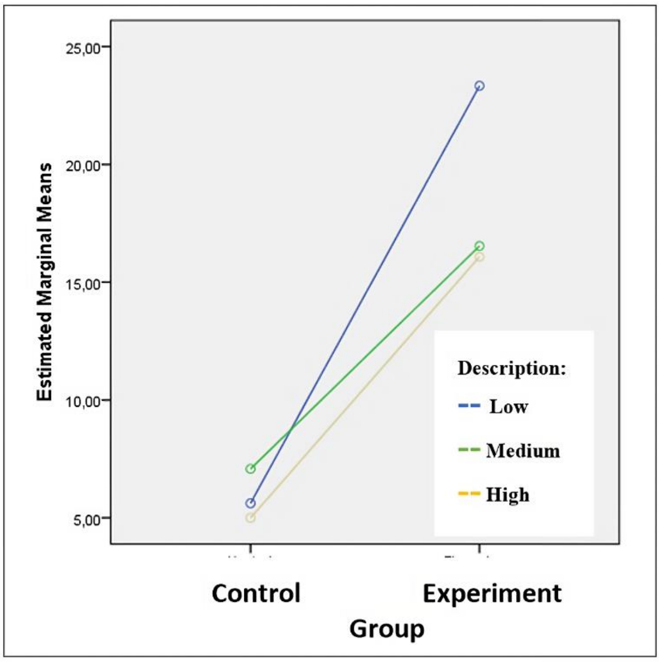

Figure 5. Interaction effect of teaching approaches & prospective teachers' IMA on improvement of their reversible thinking ability (Source: Author's own elaboration, using SPSS software)

occurs in low, moderate, and high IMA levels. Moreover, **Figure 5** shows that prospective teachers with moderate (medium), low, and high IMA levels ranked highest to lowest in terms of gain scores in the control group. Meanwhile, the same ranking was occupied by prospective teachers with low, moderate, and high IMA levels in the experimental group. The difference in the order of gain score based on the IMA level between the experimental and control groups indicates an interaction effect between the teaching approach and IMA level on improving prospective teachers' reversible thinking ability. However, before determining whether this interaction effect is significant, it must be confirmed by testing the differences in gain score between IMA levels in both the experimental and control groups.

To determine the significant difference, Kruskal-Wallis test was used. First, the test was done to all IMA levels' gain scores in the experimental group. Through the Kruskal-Wallis test, it was obtained the sig. value $= 0.055 > 0.05 = \alpha$, therefore, accept H_0. The result implies no significant difference between the low, moderate, and high IMA levels in the experimental group's reversible thinking ability gain scores. Second, the same test was conducted in the control group. The test result shows that the value of sig. was $0.238 > 0.05 = \alpha$; thus, accept H_0. The test indicates no significant difference between the low, moderate, and high IMA levels in the control group's reversible thinking ability gain scores.

According to the test results comparing the differences in gain values between IMA levels in each group, there is no significant difference between the gain values of low, moderate, and high IMA levels. Therefore, it is assumed that there is no noticeable interaction effect of the teaching approach and IMA levels on improving prospective mathematics teachers' reversible thinking ability.

DISCUSSION

The analysis results have shown that prospective mathematics teachers who received teaching by metacognitive approach significantly improved their reversible thinking ability compared to those who received teaching by direct approach. Thus, teaching mathematics to prospective teachers using a metacognitive approach improves their reversible thinking ability.

Temur et al. (2019) mentioned that the metacognitive approach could enhance students' mathematics problem-solving abilities. Their finding is in line with other studies, which suggest a positive relationship between the application of metacognitive teaching and the development of students' beliefs and problem-solving abilities (Depaepe et al., 2010; Lee et al., 2014; Shilo & Kramarski, 2019). Filling the gap that previous findings have not identified, the results of this study elaborate that a metacognitive approach can improve problem-solving ability in general and effectively improve reversible thinking ability, which is a part of problem-solving abilities.

Furthermore, reversible thinking is a necessary ability individuals require to solve mathematical problems (Saparwadi et al., 2020; Simon et al., 2016). According to Hackenberg (2010), to improve reversible thinking in solving problems, one must learn to construct schematic planning of the problem's results and engage in more mental processes that enable one to think in multiple procedures to find the solution. Therefore, this study recommends implementing the metacognitive approach to help prospective teachers practice reversible thinking in solving mathematical problems. Ultimately, this approach encourages one to formulate the problem context into mathematical models, identify the relationship between initial states/what is known and goal states/what is asked in the problem, design appropriate strategies, and evaluate the answer obtained.

In the metacognitive approach applied in this study, learning begins with assigning a task or mathematical problem to prospective teachers. When they understand the problem, they will pose conjectures or questions that aid in finding alternative solutions. However, when they experience difficulties, the lecturers guide them by asking questions that stimulate the development of more interactive thought processes and allow them to connect previously and currently learned materials. This approach demonstrates that prospective teachers can understand the problem (identify the initial and goal states), devise possible solution plans, execute the plan, and interpret and re-examine the answers they obtain.

Re-examining or evaluating the answers obtained is necessary since understanding a result in mathematics means an individual can critically validate the argument: and know not just the result but also how to be confident about it (Schaathun, 2022). The strategy of asking questions aligns with the recommendations of Smith and Sherin (2019), who stated that questioning strategies allow individuals to advance their thinking. In addition, the results of this study regarding questioning strategy are consistent with those of Kwangmuang et al. (2021), who reported that individuals, particularly students, who received problem-based learning had higher average scores on several mathematical abilities, including the ability to solve non-routine problems. Therefore, this study provides evidence that the metacognitive approach, in addition to the problem-based learning reported by previous research, could significantly improve reversible thinking, one of the problem-solving abilities required of prospective mathematics teachers.

This study has shown that teaching with a metacognitive approach improves prospective teachers' reversible thinking ability; however, we cannot assume that teaching with a direct approach is ineffective. Ku et al. (2014) and Kuhn (2007) stated that learning with direct-approach transfers knowledge explicitly from the teacher to the student. In this study, university lecturers demonstrate direct-approach teaching by explaining a concept and providing examples of the mathematical problem and its solution related to the materials discussed. Through this study, both teaching approaches are evidence to improve prospective teachers' reversible thinking ability (**Table 2**). Nevertheless, the improvement of the reversible thinking ability of prospective teachers who received teaching based on a metacognitive approach was more significant than those taught based on a direct approach. Unlike the metacognitive approach that promotes two-way communication between the lecturer and learners, the direct approach, or what should be said as 'traditional' lectures, makes learners ought to follow the instructions and have a passive role during the learning scenario (Kempen, 2021).

Apart from prospective teachers with high initial mathematics ability, it appears that implementing teaching approaches (metacognitive and direct approach) on prospective teachers' IMA levels has no significant effect on improving their reversible thinking ability. A significant improvement in the reversible thinking ability of high IMA prospective teachers who received a metacognitive approach may be subject to their decent mathematical ability foundations. As stated by Maf'ulah et al. (2017), all aspects of reversible thinking are met by individuals who can employ algorithms and have an excellent conceptual understanding. Moreover, in teaching with a metacognitive approach, the lecturer presents questions to induce one's metacognition. The high IMA level prospective teachers benefit from the lecturer's approach to independently grasp hidden information from the problem assigned, devise and employ a good solution plan, and look back to their solution procedure.

The above description highlights that the existence of a metacognitive approach by lecturers can improve the reversible thinking ability of high IMA-level prospective mathematics teachers. Likewise, with the help of high IMA-level prospective teachers, those with moderate and low-IMA levels can also improve their reversible thinking ability.

According to the result and discussion described above, this paper suggests some implications in teaching and learning mathematics for higher education. First, being proficient in reversible thinking ability is essential for prospective teachers since it can help them solve both academic (in mathematics) and real-world problems. Moreover, the recent study's findings revealed that the metacognitive approach effectively improves prospective teachers' reversible thinking in all IMA levels. Thus, it needs to be considered one of the alternative teaching approaches in higher education, especially in teacher education. Finally, addressing the importance of reversible thinking in mathematics learning, this study recommends that reversible thinking ability be included as one of the abilities attained by prospective teachers in the teacher education curriculum for future endeavors.

CONCLUSIONS

The difference in the improvement of prospective teachers' reversible thinking ability occurs between those who received a metacognitive approach and those who received a direct approach teaching. Prospective teachers receiving a metacognitive approach had significantly higher average post-test score assessing their reversible thinking ability than those who received a direct approach.

Similar results were also obtained with groups of prospective teachers with low, moderate, and high IMA. The result highlights that the average reversible thinking ability post-test scores of prospective teachers with low, moderate, and high IMA who received the metacognitive approach are significantly higher than those who received the direct approach. Therefore, it can be concluded that in all initial mathematics abilities, the metacognitive approach can improve prospective teachers' reversible thinking ability better than the direct approach.

However, there is no interaction effect of the teaching approach and the grouping of learners according to IMA to improve their' reversible thinking ability. The results are indicated by:

(1) in terms of prospective teachers who received a metacognitive approach, there is no distinction in the improvement of their reversible thinking between those with low, moderate, and high initial mathematical abilities and

(2) in terms of prospective teachers who received a direct approach, there is no distinction in the improvement of their reversible thinking between those with low, moderate, and high initial mathematical abilities.

Although some existing studies focused their research metacognitive approach or students' reversible thinking, this paper offers two important contributions to mathematics learning:

(1) previous findings identified that the metacognitive approach could improve students' problem-solving ability in general, this paper specified that the approach effectively improves students' reversible thinking ability, which is a part of problem-solving abilities and

(2) existing studies reported that reversible thinking ability is mainly can be improved by implementing the problem-based learning, this paper provides another evidence that the metacognitive approach is also valuable to improve the same ability.

Therefore, the findings of this paper suggest that a metacognitive approach should be considered as one of the alternative teaching approaches in higher education, particularly in teacher education.

Limitation

Nevertheless, this study is far from perfectly-covered research about the effectiveness of the metacognitive approach in improving prospective teachers' reversible thinking. What has been investigated and analyzed in this study is limited to fractions. Further research might extend to other mathematics topics and a joint of two or more mathematics topics to enhance prospective teachers' ability to integrate those topics into one mathematical problem. Therefore, not only practicing prospective teachers' reversible thinking but also enhancing their relational understanding of mathematics concepts.

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